PROM^2 for Girls 2025

Team Round

April 26, 2025

Problem 1

Every year, PROM^2 for Girls provides free T-shirts for participants (big thanks to our sponsors!). We always choose Custom Ink to produce these T-shirts. The unit price varies based on the number of T-shirts people place in order. The price table is attached below.

Number of T-shirts	1-20	21-50	51-100	101-200	201-500	>500
Unit Price	$14.5	$13.5	$12.5	$11.5	$10.5	$9.5

Last year, we ordered 135 T-shirts at the beginning. Later, we found that more students registered, so we place a 2nd order of 80 T-shirts. How much money would we save if we combine those two orders as one at the beginning?

Problem 2

Sophia qualified for MOP last year because she did well in USAMO, which is a test that has 6 questions. For each question, a participant could get 0, 1, 2, 3, 4, 5, 6, or 7 credits. If the cutoff for qualifying for the next level, MOP, is 37, what is the least amount of questions Sophia needed to get full credit to qualify for MOP?

Problem 3

PRISMS students all love ice cream! For a party, Elisa, the MC of PROM^2, wants to get 3 gallons of ice cream for all the volunteers. The ice creams come in three sizes: pints, quarts, and gallons. If there are four quarts in a gallon and two pints in a quart, how many ways can she choose the sizes of the ice cream to get 3 gallons of ice cream?

Problem 4

Find all the solutions to the equation

$$\frac{\max(21,x)+\min(9,x)}{\max(9,x)-\min(3,x)}=\frac{17}{13},$$

where max(a,b) is denoted as the larger number between a and b, and min(a,b) as the smaller number.

Problem 5

27 unit cubes are glued together to form a larger cube. Some of the faces of the large cube are painted red and others faces are painted blue. Denote R as the number of unit cubes that at least one face is painted red, and B as the number of unit cubes that at least one face is painted blue. Find the maximum possible value of R+B.

Problem 6
Jessie draws a triangle *ABC* with side lengths 3, 4, 5. She finds that the three angle bisectors are concurrent at a point *P* in the triangle. Let G_1, G_2, G_3 be the centroids of the triangles *PAB*, *PAC*, and *PBC*, respectively. What is the area of the triangle $G_1G_2G_3$?
(The intersection of three angle bisectors of a triangle is the incenter of the triangle, and the intersection of three medians of a triangle is the centroid of the triangle.)

Problem 7
Catherine colored 30 points red in the coordinates plane, where none three of those red points are colinear. She connected some pairs of red points by line segments. For each red point, define the degree of this point as the number of line segments connecting to it. Catherine labelled each red point with the square of its degree, and labelled each line segment with the sum of the degrees of its two endpoints. She subtracted the sum of numbers at all red points by the sum of numbers at all line segments. What is the maximum possible results Catherine could get?

Problem 8
Your origami instructors, Jessie and Melinda, start to read the book <The Book Thief> on the same day. Melinda reads 16 pages every day. Jessie reads 5 pages on the first day. From the 2nd day to the 15th day, each day she reads 2 more pages than she does on the previous day (thus, she reads 33 pages on the 15th day). Since the 16th day, she starts to prepare for the Origami Tournament and each day she reads 2 less pages than she does on the previous day. On the last day, she reads 1 page and finishes the entire book. By the end of which day, the number of pages that Jessie has read more than Melinda has is maximized?

Problem 9
Find all the positive integers $a < b$ such that $a^3 + (a+1)^3 + (a+2)^3 + \cdots + b^3 = 12345678$.

Problem 10
Let *n*>3 be a positive integer. The sum of the measures of some (not all) interior angles of a regular polygon with *n* sides is 3300. Find all the possible values of *n*.