PROM^2 for Girls 2024

PRISMS Math Team

April 2024

1. Evaluate $(2+0+2+4)\cdot(2^2+0^2+2^2+4^2)$.

   (A) 64 (B) 128 (C) 160 (D) 192 (E) 240

2. The student government had a bake sale every day from April 15th to April 21st. Every day, they sold at most 15 cupcakes. They end up selling 100 cupcakes by the end of the week. What is the least number of cupcakes they sold on any day?

   (A) 5 (B) 8 (C) 10 (D) 12 (E) 14

3. Let N be the smallest positive integer that gives the following remainders: 1 when divided by 2, 2 when divided by 3, 3 when divided by 4, 4 when divided by 5, 5 when divided by 6. Which of the following intervals does N lie in?

   (A) $[50,60)$ (B) $[70,80)$ (C) $[110,120)$ (D) $[120,130)$ (E) $[710,720)$

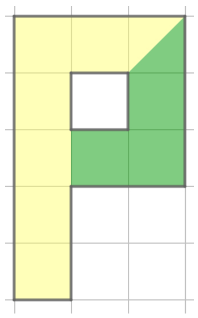
4. Evaluate
$$\frac{2024-20.24}{33}.$$

   (A) 11.18 (B) 20.24 (C) 60.72 (D) 62 (E) 66

5. Given that the 3-digit integer $\overline{3A7}$ is multiple of 9, find the value of A. (For example, if $A=0$, $\overline{3A7}=307$.)

   (A) 1 (B) 2 (C) 5 (D) 8 (E) 9

6. As shown below, the letter P has been divided into two regions. Given that each square grid has side length 1, what is the difference between the areas of the yellow region and the green region?



   (A) 2 (B) 3 (C) 3.5 (D) 4 (E) 6.5

7. There are n points in the plane. Every two of them are connected with a line segment. What is the maximum value of n such that all of the segments do not intersect each other except for at the vertices?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

8. 7 students, Asel, Ben, Cedric, David, Eva, Felicity, Grace sit around a round table clockwise in that order. Asel calls out 1, Ben calls out 2, Cedric calls out 3, and the rest continue repeating calling out 1, 2, then 3. When someone says 3, they will leave the table immediately and the counting continues clockwise. After they continue this process for a while, there is only one person still sitting at the table. Who is it?

(A) Asel (B) Ben (C) David (D) Eva (E) Grace

9. An arithmetic sequence is a sequence of numbers such that the difference from any succeeding term to its preceding term remains constant throughout the sequence. An arithmetic sequence has its first four terms $a, 2a, b, 6-b-a$. Find its 100th term.

(A) $\frac{100}{3}$ (B) 75 (C) 100 (D) $\frac{400}{3}$ (E) 300

10. Wendy is shooting an arrow on a board with a rectangular target of size 14in $\times$ 34in. Given that her arrow lands randomly on any point on the board with equal chance, and the probability the arrow hits the target is 17%, what is the total area of the board not inside the target (in in^2)?

(A) 476 (B) 924 (C) 1400 (D) 2324 (E) 2800

11. The math team is holding a party. There are n participants, and every two of them are either friends or strangers, What is the minimum possible number of participants, such that there must be at least one group of three people, such that either every two of them are strangers, or every two of them are friends?

(A) 4 (B) 5 (C) 6 (D) 8 (E) 9

12. Felicity was taking Honors Biology. The final grade consists of 40% semester 1 score, 40% semester 2 score, and 20% project score. A grade greater than or equal to 93 is an A. If she got 88 in semester 1, what's the minimum score she needs in semester 2 to get an A in final grade? (The possible scores are integers from 0 to 100)

(A) 92 (B) 93 (C) 94 (D) 95 (E) 98

13. While the $PROM^2$ Academy is preparing lunch, the staff realize that the number of eggs is one less than a multiple of 12, as indicated by the dozen boxes. However, when they arrange the eggs into rows of 7, there is none left. What interval does the minimum number of eggs possible lie?

(A) $[20, 30)$ (B) $[30, 40)$ (C) $[40, 50)$ (D) $[50, 60)$ (E) $[60, 70)$

14. There are 100 towns in $PROM^2$ City, and 98 roads are built among them. Each road connects two towns, and there is at most one road between each pair of towns. A "group" is a set of towns, such that two towns are in the same group if and only if one can go from one town to another through one or more roads. What is the minimum possible number of groups in $PROM^2$ City?

(A) 2 (B) 3 (C) 50 (D) 98 (E) 99

15. Annie has a bunny named Pumpkin. Annie's friends, Amy, Olivia, and Sophia, all adore Pumpkin and like to bring carrots and lettuce for Pumpkin every few days. Amy visits once every 4 days, Olivia visits once every 5 days, Sophia visits every Wednesday and Saturday of the week. Pumpkin's birthday is on January 1st, a Wednesday, and Amy, Olivia, and Sophia all visit on that day. In the month of January, how many days will at least one of Annie's friends come visit pumpkin? (January has 31 days).

(A) 5 (B) 17 (C) 18 (D) 19 (E) 23

16. In the card game *Egyptian Rat Screw*, the pile of played cards can be slapped when either: (1) the top 2 cards are of the same rank, or (2) the top card is of the same rank as the third from the top. Right now the pile is empty, and David keeps adding cards to the top of the pile. Given that each card has an integer rank between 2 and 10, inclusive, how many different combinations of ranks are there such that when the third card is added, the pile can be slapped for the first time?

(A) 72 (B) 81 (C) 90 (D) 144 (E) 162

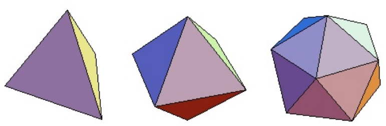
17. There are four townships in $PROM^2$ City, named Alpha, Bravo, Charlie, and Delta. Given that Alpha is 12 miles away from Bravo, Bravo is 8 miles away from Charlie, and Charlie is 6 miles away from Delta. Which of the following is not a possible distance, in miles, between Alpha and Delta?

(A) $\frac{\sqrt{2}}{\pi^{\pi}}$ (B) 6 (C) 12 (D) 20 (E) 30

18. Denote $\sigma(n)$ as the sum of all the divisors of n. For example, $\sigma(12) = 1+2+3+4+6+12 = 28$. Evaluate $\sigma(960) - \sigma(480)$.

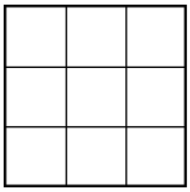
(A) 480 (B) 960 (C) 1440 (D) 1536 (E) 1920

19. Alice has 52 equilateral triangles. She could only stick 4 together to form a tetrahedron (4 vertices), or 8 together to form an octahedron (6 vertices), or 20 together to form an icosahedron (12 vertices). Given that she needs to use all of the triangles she had, what is the minimum number of the sum of the number of vertices?



(A) 22 (B) 30 (C) 34 (D) 40 (E) 52

20. David has to mop the floor with 9 square cells as shown below. He can begin mopping in any cell and then move to an adjacent cell in every turn. (Two cells are adjacent if and only if they share an edge.) In how many ways can David mop the entire floor without revisiting any cell?



(A) 8 (B) 24 (C) 40 (D) 48 (E) 64

21. Find the greatest radius of a circle inscribed in a triangle inscribed in the unit circle.

(A) $\frac{1}{3}$ (B) $\sqrt{2}-1$ (C) $\frac{\sqrt{3}}{3}$ (D) $\frac{1}{2}$ (E) $\frac{\sqrt{6}}{2}$

22. Find the number of ways to tile a 3×10 board with 3×1 or 1×3 tiles.

(A) 24 (B) 28 (C) 32 (D) 40 (E) 42

23. Integers x, y, z, w satisfy $3^{x+2} - 3^{y+0} + 3^{z+2} - 3^{w+4} = 2024$. Find $x + y + z + w$.

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

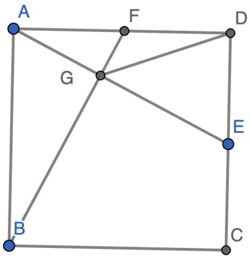
24. Atticus and Ray plan to meet in a library. Normally, it will take Atticus 40 minutes to reach there and Ray 30 minutes. However, on each person's way there is extraordinarily congested traffic. It will consume Atticus m extra minutes and Ray n extra minutes, in which m is a random real number chosen uniformly from the interval $[0, 30]$ and n a random number chosen uniformly from the interval $[0, 40]$. If they leave their homes simultaneously, find the possibility of Ray waiting for Atticus, but for a duration of less than 20 minutes.

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{3}{8}$ (D) $\frac{11}{24}$ (E) $\frac{13}{24}$

25. In summer, Elmo stayed at Grover's house. Elmo found that Grover went to library to volunteer every 4 days and went swimming every 3 days. Elmo remembered that Grover went to library to volunteer on the first day of his arrival and went swimming on the last day of his stay. If Grover went to library to volunteer and went swimming on the same day four times during Elmo's stay, at most how many days Elmo stayed at Grover's house this summer?

(A) 37 (B) 49 (C) 54 (D) 66 (E) 71

26. As shown in the diagram, $ABCD$ is a square with side length 2. E is a point that moves freely on the segment CD, and F lies on AD such that BF is perpendicular to AE. What is the minimum possible length of DG?



(A) 1 (B) $\sqrt{5}-1$ (C) $\sqrt{2}$ (D) $\frac{3}{2}$ (E) $\sqrt{3}$

27. Positive integers x, y, z, w satisfy $xy - zw = 1000$. Find the minimum possible value of $x+y+z+w$.

    (A) 68 (B) 70 (C) 72 (D) 74 (E) 76

28. PRISMS students hold PROM (Dance Party) every May. Last year in the PROM, every girl danced with 5 boys and 3 girls, while every boy danced with 4 girls and 4 boys. If 360 pairs of students danced at the party, how many girls attended the PROM?

    (A) 20 (B) 25 (C) 40 (D) 50 (E) 60

29. Eva wrote a number with two decimal digits in base-6: $(0.\overline{xy})_6$ on a blackboard. For example, if she wrote $(0.14)_6$, then the number equals $\frac{1}{6}+\frac{4}{6^2}$. Spencer converted it to a repeating decimal in base-7: $(0.\dot{z}\dot{w})_7$. Victor noticed that $x+y=z+w$, and he converted it to base-10. Find the sum of the different digits appearing in this number's base-10 representation. (If a number appears in the repeating unit, it is only added once)

    (A) 11 (B) 14 (C) 16 (D) 17 (E) 20

30. We know $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6}$. Find $\frac{1}{1^2\cdot 2^2}+\frac{1}{2^2\cdot 3^2}+\frac{1}{3^2\cdot 4^2}+\ldots$.

    (A) $\frac{\pi^2}{6}-\frac{3}{2}$ (B) $\frac{\pi^2}{3}-3$ (C) $\frac{\pi^4}{36}$ (D) $\frac{\pi^4}{36}-\frac{\pi^2}{6}$ (E) $\frac{\pi^2}{12}$

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